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# Chaotic characteristics of elastic waves propagating in concrete material

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## ABSTRACT

In this study, chaotic properties of elastic waves transmitted through concrete material were experimentally evaluated. The elastic waves were introduced by ultrasonic probe and the received signals were analysed to determine several chaotic parameters. As a result, it was found that the first 0.6msec of Coda waves has chaotic characteristics and the density of aggregates and W/C of concrete may affect some chaotic parameters of obtained signals.

**Keywords.** Chaotic behaviour, Lyapunov exponent, correlation dimension, elastic wave

## INTRODUCTION

The elastic waves propagating in concrete material scatter, diffuse and reflect at the aggregates or voids which have different acoustic impedance, and they are interfered and multiplied with each other over the travel time. Thus the diffused waves have travelled much longer paths through the medium than the first arrival waves due to multiple scattering (Becker et al., 2003). This phenomenon can be regarded as Coda wave which is one of the strong none-linear phenomena (Dennis, 200). In Coda wave, small change of the medium results in large changes in the multiplied waves due to strong none-linearity of the medium. Due to its strong non-linearity, the evaluation of Coda wave could be an alternative approach to evaluate the concrete material, especially concerning meso-scale structures such as porosity or weak interface around the aggregates.

On the other hand, chaotic signal is also caused by strong none-linearity of the system. When the initial condition is slightly different in the system with strong none-linearity, the signal is drastically different with sufficient time due to none-linearity. This analogy may lead to the idea that the Coda wave in concrete may have some chaotic characteristics.

Therefore, in this study, chaotic characteristics of the transmitted elastic waves through concrete material are evaluated. Especially, the effect of aggregates on the chaotic behaviour is clarified because the aggregates may affect the scattered waves in concrete. Herein we evaluate the degree of chaos using Lyapunov exponent, correlation dimension and also prediction error based on attractor.

## CHAOTIC CHARACTERISTICS

**Attractor**. In a dynamical system with k-dimension, the state change of the system can be described using $k$ state variables. Attractor is a behaviour or orbit of the system in the state space of $k$ dimensions. Normally in practical cases, only one variable can be observed from the multi-dimensional system, and it is difficult to construct the orbit in the state space with original dimension. F.Takens (1981) proves that the state vector $\boldsymbol{v}(t)$with $m$ dimensions given by the equation (1) can be reconstructed from the single variable when $m$ is greater than $2k + 1$.

$$\boldsymbol{v}(t) = (y(t), y(t+\tau), \dots, y(t+(m-1)\tau)) \tag{1}$$

where $y(t)$ is the observed signal and $\tau$ is the time delay. Thus in this study, the attractor of the transmitted waves in concrete is reconstructed with appropriate dimensions and delay time. Note that generally it is difficult to determine appropriate dimension and delay for the implicit system where dominant equation is not explicitly described. The elastic wave propagating in concrete medium is also "implicit system" and it is difficult to determine these values. We assumed these values so that maximum Lyapunov exponent becomes positive and it reaches its maximum.

**Lyapunov exponent**. Strong none-linearity may cause unstable behaviour over long term span. This characteristic appears in attractor as diffusion of closest two points in different orbits. The expansion ratio of these two points can be expressed as exponent function and this exponent is known as Lyapunov exponent. When Lyapunov exponent is positive, the system can be regarded as chaos. In this study, Lyapunov exponent of the signal is evaluated. We calculate Lyapunov exponent on the basis of Kantz method (Kantz, 1994). In this method, the ratio of expansion of the orbits is obtained by the average distance of nearest points, $S(\tau)$, which is given by

$$S(\tau) = \frac{1}{\mathrm{N}} \sum_{t=1}^{N} \log\left( \sum_{k_i=1}^{M} \boldsymbol{d}(\boldsymbol{v}(t), \boldsymbol{v}(k_i); \tau) \right) \tag{2}$$

where $\boldsymbol{d}(\boldsymbol{v}(t), \boldsymbol{v}(k_i); \tau)$ is the distance between $\boldsymbol{v}(t)$ and its $\varepsilon$-nearest points $\boldsymbol{v}(k_i)$ after the time of $\tau$, i.e.,

$$\boldsymbol{d}(\boldsymbol{v}(t), \boldsymbol{v}(k_i); \tau) = |\, y(t+\tau) - \; y(k_i+\tau)| \tag{3}$$

Then the maximum Lyapunov exponent can be obtained by slope of $S(\tau)$ with respect to $\tau$.

**Correlation dimension.** Correlation dimension is one of the fractal dimensions and there are several methods to estimate the dimension from time series. In this study, GP method is adopted for the estimation. In GP method, the dimension can be calculated by the slope of correlation integral, $C^m(r)$, given by

$$C^m(r) = \lim_{N \to \infty} \sum_{\substack{i,j=1 \\ i \neq j}}^{N} I\left(r - \left|\boldsymbol{v}_i - \boldsymbol{v}_j\right|\right) \tag{4}$$

where $I$ is the Heaviside function in the form

$$I(t) = \begin{cases} 1 & (t \geq 0) \\ 0 & (t < 0) \end{cases}. \quad (5)$$

This integral indicates the ratio of the points of attractor which are inside the $m$-dimensional hypersphere center on $\boldsymbol{x}_i$ ($i = 1,2, \ldots, \mathrm{N}$). The slope of $C^m(r)$ with respect to radius $r$ in the log-log scale is the correlation dimension.

**Prediction error.** From the deterministic chaos theory, prediction of time series can be attained because the phenomenon is deterministic and the attractors of two time series are similar to each other when these time series have similar chaotic characteristics. Now we have a time series as a supervisor and reconstruct the attractor from this supervisor with appropriate dimension and delay. Then the time series to be estimated is also plotted in the same state space as the supervisor. After by selecting the nearest point of the supervisor, $\boldsymbol{x}(t)$, to the point of the data to be estimated, $\boldsymbol{z}(t)$. By following the attractor orbit of the supervisor from the point of $\boldsymbol{x}(t)$, the prediction vector, $\mathrm{d}\boldsymbol{x}(t) = \boldsymbol{x}(t+1) - \boldsymbol{x}(t)$, can be formed. Finally the next point to $\boldsymbol{z}(t)$ in the data can be obtained by simply adding the prediction vector to the point in the form

$$\hat{z}(t+1) = \boldsymbol{z}(t) + \mathrm{d}\boldsymbol{x}(t). \quad (6)$$

Thus the prediction error is the error between actual value of $\boldsymbol{z}(t+1)$ and predicted value, $\hat{\boldsymbol{z}}(t+1)$, as given by$\Delta(t) = |\boldsymbol{z}(t+1) - \hat{\boldsymbol{z}}(t+1)|$.

## EXPERIMENTAL DESCRIPTION

**Specimens.** In this study, we mainly focus on the effect of aggregates in concrete. Iwanami et al. (1999) pointed out that velocity of elastic wave in concrete strongly depends on the quality of matrix such as W/C but slightly on the aggregates. However, center frequency of the elastic waves is related to the geometrical features of aggregates. Thus in this study cylindrical and cubic specimens are fabricated with several composition of aggregates as listed in Table 1. The specimen used in this study is cubic specimen with a dimension of 100mm x 100mm x 400mm. For the specimens of W/C=30%, plasticiser is added in mixing water and that of 65% segregation resistant is also added to prevent from segregation due to high W/C.

**Table 1. Series of specimens**

| W/C | G-max | Density of aggregates |
|---|---|---|
| 50% | 15mm | 55% |
| 50% | 15mm | 68% |
| 50% | 15mm | 78% |
| 30% | 15mm | 68% |
| 65% | 15mm | 68% |

**Measurement system.** In this study, function generator and amplifier is used for initiating the wave on concrete and the received wave is also amplified and recorded logger as shown in Figure 1. The function generator gives a pulse with 10μ sec width at every 0.1 sec and amplified by 20 times up to 200Volt. The pulse signal is also recorded directly from the generator and the signals are recorded at 5M Hz with a length of 50m sec. The probe has resonant frequency of 40 kHz and this characteristic may affect the result. However, throughout the test we used the identical probe and this effect can be ignored when we evaluated the data relatively.

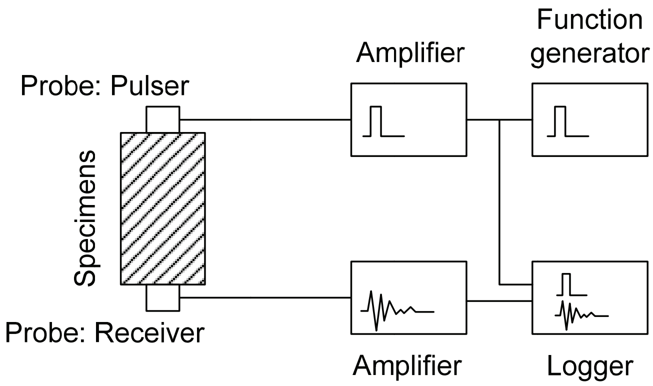


**Figure 1. Measurement set up**

**Data.** The waves propagating in concrete can be regarded Coda wave and thus we focus on the data from $2t_s$ where $t_s$ is the arrival time of S wave because after $2t_s$ the waves are thought to be sufficiently superposed. Now we have two sections of time, each of which is 0.6 msec, and chaotic characteristics are calculated in each section in order to decrease the effect of damping. One example is shown in Figure 2. Note that the signal in each section is normalized so that maximum and minimum values match 1 and -1, respectively.

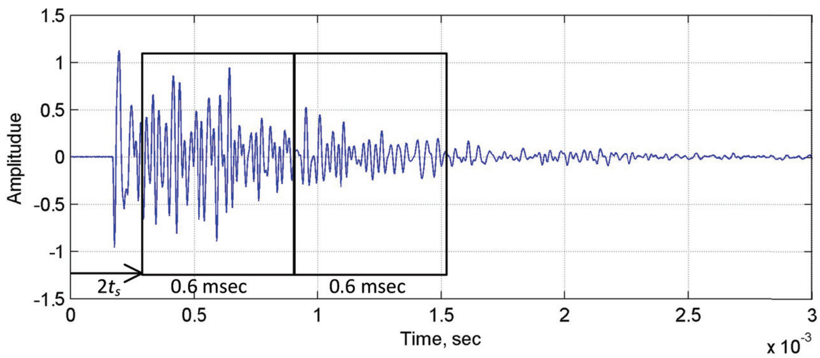


**Figure 2. Example of obtained signals and analysed section**

## RESULTS AND DISCUSSION

Figure 3 and Figure 4 shows the relationship between several indices and aggregate density when W/C=50%, Gmax=15mm. Note that upper figures in Figure 3 show the chaotic indices vs. aggregate density in the first time section of 0 to 0.6 msec, while lower figures show those in the next section of 0.6 to 1.2 msec. From these figures, it is found that maximum Lyapunov exponent and also correlation dimension in the first section decreases as the density increases. However, the prediction error has no clear relation to the density.

Moreover, in the second section even the Lyapunov and dimension do not have clear relation to the density. Lyapunov exponent may indicate the strength of chaotic behaviour and thus the increase of aggregate may decrease the chaotic property. As for the correlation dimension, when the dimension becomes integer the geometry of the attractor loses its fractal property, which means the chaotic property may also decreases as the dimension reaches the integer number. In the second section, Lyapunov exponent becomes larger than that in the first section and also the dimension reaches about 1.4. This may indicate that chaotic property in the second section is stronger than that in the first section because sufficient superposition of reflected waves from the aggregates yields chaotic behaviour in the second section. The effect of aggregate density may disappear or reach its upper bound when the waves are sufficiently superposed.

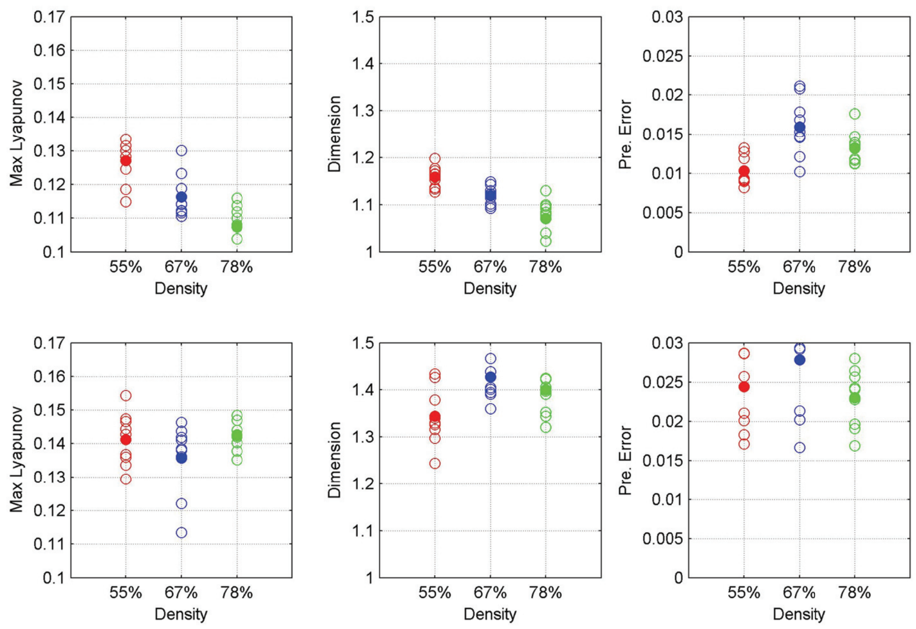


**Figure 3. Relationship between chaotic indices and density of aggregates (Gmax=15mm): time section from 0 to 0.6msec in upper figures and that from 0.6 to 1.2 in lower figures.**

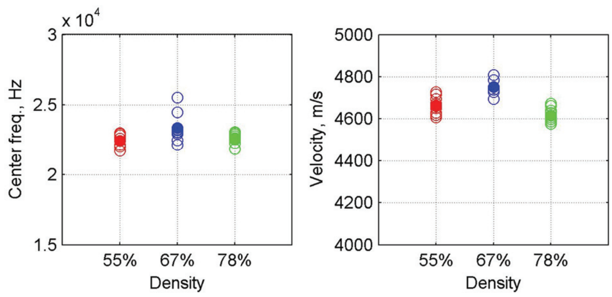


**Figure 4. Relationship between density of aggregates (Gmax=15mm) and center frequency and velocity.**

On the other hand, the center frequency as well as velocity has almost no relation to the density. The effect of aggregate density may not affect the velocity which is also confirmed in the other researches, but the center frequency may have some relation to the density of aggregates, which is not confirmed in this research. The dominant frequencies of the input signal are almost 20-40 kHz and the G max of aggregates is 15mm which is equal to the wavelength of few hundred kHz, which may not affect the waves having the frequency range of 20-40 kHz. Thus the cneter frequency of waves is almost identical in every density.

Figure 5 shows the relationship of the indices and W/C when the density is 65%, Gmax=15mm. The upper figures show the Lyapunov exponent, the dimension and prediction error in the first section and the lower figures show the center frequency and velocity of the waves. Note that the indices do not have clear relation to W/C in the second section. From this figure, it is found that the Lyapunov exponent and dimension may slightly increase as W/C increases, but prediction error may not have clear relation to W/C, and that the velocity strongly depends on W/C but center frequency does not. This may be attributed to the fact that low quality of matrix in the concrete may lead to damping of waves and this yield the same effect of long propagating path which makes sufficient superposition. However, this trend may be differed when the characteristic of input wave such as dominant frequency is changed. Of course, the velocity decreases as W/C increases due to the low quality of matrix.

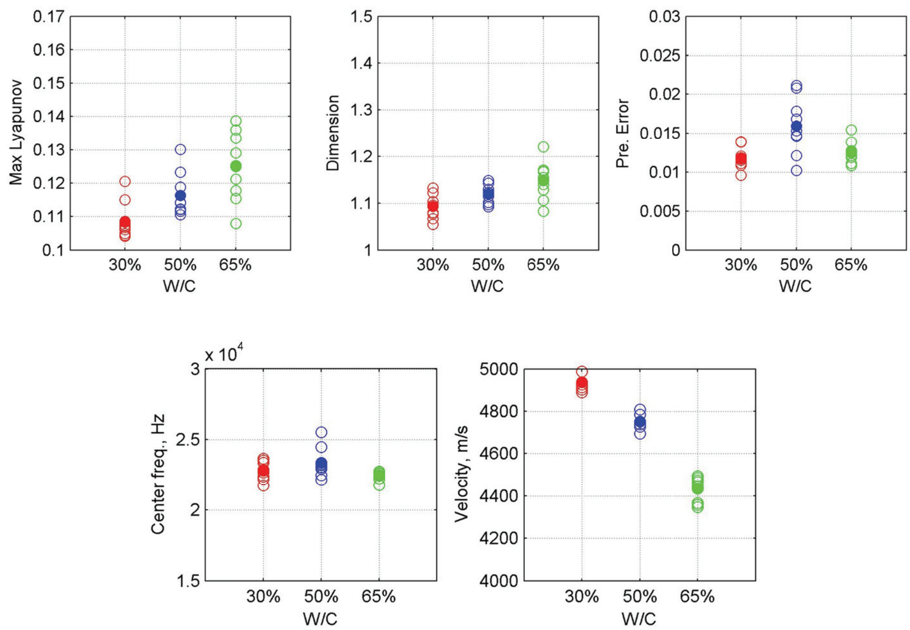


**Figure 5. Relationship between W/C and chaotic indices in the first time section as well as center frequency and velocity (Gmax=15mm, density=65%).**

## CONCLUSION

In this study, chaotic characteristics of the transmitted elastic waves through concrete material are evaluated. Especially, the effect of aggregates on the chaotic behaviour is clarified. As a result, it is found that the density of aggregates as well as W/C may affect the Lyapunov exponent and also correlation dimension but these effects may disappear but chaotic behaviour becomes strong when the elapsed time increases. This result may open up the possibility of quality evaluation for concrete material such as porosity and weak interface around the aggregates. These trends, however, may be different when the characteristic of input wave is different, and thus the characteristic of input wave should be controlled or the effect of input waves should be clarified in the further study.

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