Fifth International Conference on Sustainable Construction Materials and Technologies. http://www.claisse.info/Proceedings.htm

SENSITIVITY OF THE VARIOUS PARAMETERS IN THE PREDICTION OF THE VOIDS RATIO OF MIXES WITH FINE AND COARSE PARTICLES ACCORDING TO DEWAR'S MODEL

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ABSTRACT

Particle packing models (PPMs) have been developed and evolved since 1929 and allow to formulate the proportioning of concrete mix constituents with the minimal voids ratio. In the past, this tool has led to important innovations in concrete technology, as the development of ultra-high performance concrete. Also with regard to the design of ecological concrete, PPM is a promising technique.
In our ongoing research on the design of ecological concrete incorporating recycled concrete aggregates (RCAs), the model of Dewar will be applied. This theory is however mainly developed and validated for concrete mixtures with natural aggregates. In order to validate the theory for RCAs and understand the effect of variability on the aggregate's main characteristics (e.g. voids ratio and mean size) on the voids ratio diagram, a sensitivity analysis has been performed and is reported in the paper. It includes a sensitivity in the prediction of U_n (= voids ratio of mixes of fine and coarse aggregates) and n (= fine fraction content) to the various parameters in the model of Dewar (U_1 = voids ratio of the fine fraction, U_0 = voids ratio of the coarse fraction, r = size ratio, m = spacing factor, k_{int} and k_p = empirical factors for determining the notional width factor). The results are graphically presented for one specific example and show that variation of U_0 affects mainly the U_n and n values of changing point B while variation on U_1 affects mainly the U_n value of point E and the n value of point B and point C. With regard to the size ratio, the effect of a variation on r depends on the size ratio range considered. Variation on m has a larger impact on n values than U_n values at the changing points and variation on the k_{int} results in a variation of U_n and n values, mainly for point B and C. The effect of variations of k_p on U_n and n is limited.

Keywords: particle packing model, concrete mix design, recycled concrete aggregate, sensitivity analysis, ECO-concrete

1 INTRODUCTION

The particle theory of Dewar (Dewar 1999) relies basically on the fact that when two groups of particles with different sizes (fine and coarse particles) are mixed together, the smaller will fill the voids between the larger particles. This effect will be disturbed by particle interference creating additional voids. These interactions are called the loosening effect and the wall effect and are represented in Figure 1, with cubic materials (coarse fraction with mean size D_0 and voids ratio U_0 and fine fraction with parameters D_1 and U_1) to keep it comprehensive. Due to the loosening effect, the voids ratio of the coarse particles will increase (dilation of the coarse particles by the inserted fines) and the voids ratio of the mixture will normally decrease due to the filling effect of the fines. The voids ratio of the fine particles will simultaneously be increased due to the wall effect induced by the coarse particles. In Dewar's model, the dilation effect is represented by the m-value while the wall effect is represented by the Z-value.

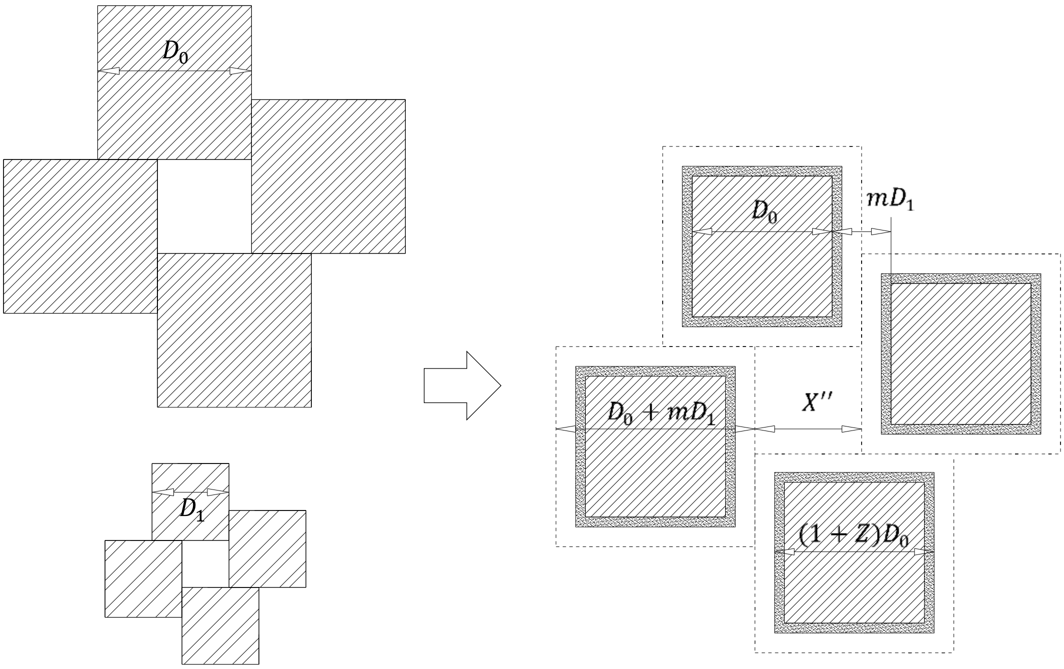

Figure 1: Schematic representation of the loosening effect and the wall effect

To calculate the voids ratio of the mixtures of fine and coarse aggregate U_n (Formula 1), the theory assumes for the fine particles an effective voids ratio of U_1'' (Formula 2) and for the dilated coarse particles an effective voids ratio of U_0'' (Formula 3).

Voids ratio of a mixture of fine and coarse particles:

$$U_n = nU_1'' \quad (1)$$

Effective voids ratio of the fine fraction:

$$U_1'' = \frac{(1+U_1)U_0''}{(1+U_0'')-(1+Z)^3} - 1 \quad (2)$$

Effective voids ratio of the coarse fraction:

$$U_0'' = (1 + U_0)(1 + mr)^3 - 1 \quad (3)$$

With

n = fraction of fines in the mixture = $\frac{U_0''}{1+U_1''+U_0''}$ (4)

r = size fraction of the particles = D_1/D_0

m = spacing factor related to the fraction of fines in the mixtures (Table 1)

Z = notional width factor = $k_{int} + \left[(1 + U_0)^{1/3} - 1 - k_{int}\right]r^{k_p}$ (5)

k_{int} and k_p are empirical factors for determining the notional width factor (see Table 1).

Dewar stated that the voids ratio diagram can be drawn by using several change points (A to F) which are associated with a particular m value and empirical factor k_{int} and k_p (Table 1). A linear function is assumed between the different change points.

Table 1: Values proposed by Dewar for the parameters m, k_{int} and k_p at the changing points.

changing points	m	k_{int}	k_p
A (n=0)	0	-	-
B	0.3	0.12	0.60
C	0.75	0.06	0.65
D	3	0.015	0.8
E	7.5	0	0.9
F (n=1)	∞	-	-

In Figure 2 an example of a voids ratio diagram is drawn for the combination of two granular materials with respective particle size D_0 and D_1 and voids ratio U_0 and U_1.

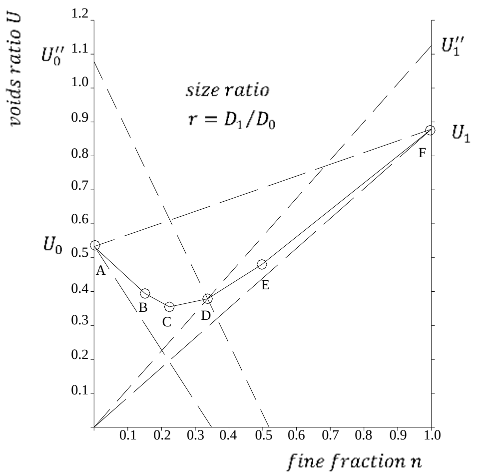

Figure 2: Exemplary theoretical voids ratio diagram of two mixed fractions

The extended theory of particle mix design of Dewar is mainly developed and validated for concrete mixtures with natural aggregates. As in the future research, ECO concrete containing recycled concrete aggregates will be designed, making use of the particle packing model of Dewar, a validation of the model is essential. To start, a sensitivity analysis is executed and reported in this paper. This information is furthermore important and very useful to quantify the effect of variation of material's characteristics U_0, U_1 and r on particle packing of mixtures. Results on variability of the material's characteristics for a range of RCAs and its effects on the voids ratio of mixtures will be presented during the conference, along with a study on the validation of Dewar's model for use of RCA.

2 SENSITIVITY ANALYSIS

2.1 Sensitivity Coefficients

To perform the sensitivity analysis, Formula (2) till (5) were incorporated in Formula (1) in order to express U_n in function of the six basic parameters (U_1, U_0, r, m, k_{int} and k_p) (Formula 6 and 7). Subsequently, the partial derivative of U_n towards each of these parameters has been determined and the sensitivity coefficients are mathematically calculated according to Formula

(8). These sensitivity coefficients express the relative change in U_n upon a relative change in one of the parameters (U_1, U_0, r, m, k_{int} and k_p). A value of one means that the relative change in this parameter is equal to the relative change in U_n.

$$U_n = nU_1'' = n\left[\frac{(1+U_1)\{(1+U_0)(1+mr)^3-1\}}{(1+\{(1+U_0)(1+mr)^3-1\})-\left(1+\left\{k_{int}+[(1+U_0)^{1/3}-1-k_{int}]r^{k_p}\right\}\right)^3}-1\right] \text{ with} \quad (6)$$

$$n = \frac{U_0''}{1+U_1''+U_0''} = \frac{[(1+U_0)(1+mr)^3-1]}{1+\left[\frac{(1+U_1)\{(1+U_0)(1+mr)^3-1\}}{(1+\{(1+U_0)(1+mr)^3-1\})-\left(1+\left[k_{int}+[(1+U_0)^{1/3}-1-k_{int}]r^{k_p}\right]\right)^3}-1\right]+[(1+U_0)(1+mr)^3-1]} \quad (7)$$

$$\frac{\partial U_n}{\partial U_1}\frac{U_1}{U_n};\ \frac{\partial U_n}{\partial U_0}\frac{U_0}{U_n};\ \frac{\partial U_n}{\partial r}\frac{r}{U_n};\ \frac{\partial U_n}{\partial m}\frac{m}{U_n};\ \frac{\partial U_n}{\partial k_{int}}\frac{k_{int}}{U_n};\ \frac{\partial U_n}{\partial k_p}\frac{k_p}{U_n} \quad (8)$$

Furthermore, the partial derivatives of n towards the same six basic parameters have been calculated together with the corresponding sensitivity coefficients (Formula 9).

$$\frac{\partial n}{\partial U_1}\frac{U_1}{n};\ \frac{\partial n}{\partial U_0}\frac{U_0}{n};\ \frac{\partial n}{\partial r}\frac{r}{n};\ \frac{\partial n}{\partial m}\frac{m}{n};\ \frac{\partial n}{\partial k_{int}}\frac{k_{int}}{n};\ \frac{\partial n}{\partial k_p}\frac{k_p}{n} \quad (9)$$

Both the sensitivity analyses to U_n and n provide useful information on the vertical and horizontal changes that may be expected in the voids ratio diagram (as shown in Figure 2) upon a change in the value of one these parameters. In the following sections, the parameters are discussed one by one and the sensitivity of U_n and n to that specific parameter is graphically presented for an example in which the other parameters are kept constant. The example considers $U_1 = 0.65$, $U_0 = 0.8$, r = 0.25 and m, k_{int} and k_p according to the values mentioned in Table 1 for the changing points B, C, D and E.

2.1.1 Voids Ratio of the Fine Fraction U_1

Since the notional width factor Z and the effective voids ratio of the coarse fraction U_0'' are independent of the voids ratio of the fine fraction U_1, not only $\frac{\partial U_n}{\partial U_1}$ and $\frac{\partial n}{\partial U_1}$ but also the corresponding sensitivity coefficients can be easily calculated (Formula 10 - 11).

$$\frac{\partial U_n}{\partial U_1}\frac{U_1}{U_n} = \frac{(1+U_0)(1+mr)^3U_1}{(1+U_0)(1+mr)^3U_1-[(1+U_1)-(1+Z)^3]} + \frac{U_1}{(1+U_0)(1+mr)^3+[(1+U_1)-(1+Z)^3]} \quad (10)$$

$$\frac{\partial n}{\partial U_1}\frac{U_1}{n} = \frac{-U_1}{(1+U_0)(1+mr)^3+(1+U_1)-(1+Z)^3} \quad (11)$$

Results of the sensitivity analysis are presented in Figure 3, for the specific case as mentioned above and for the change points B, C, D and E.

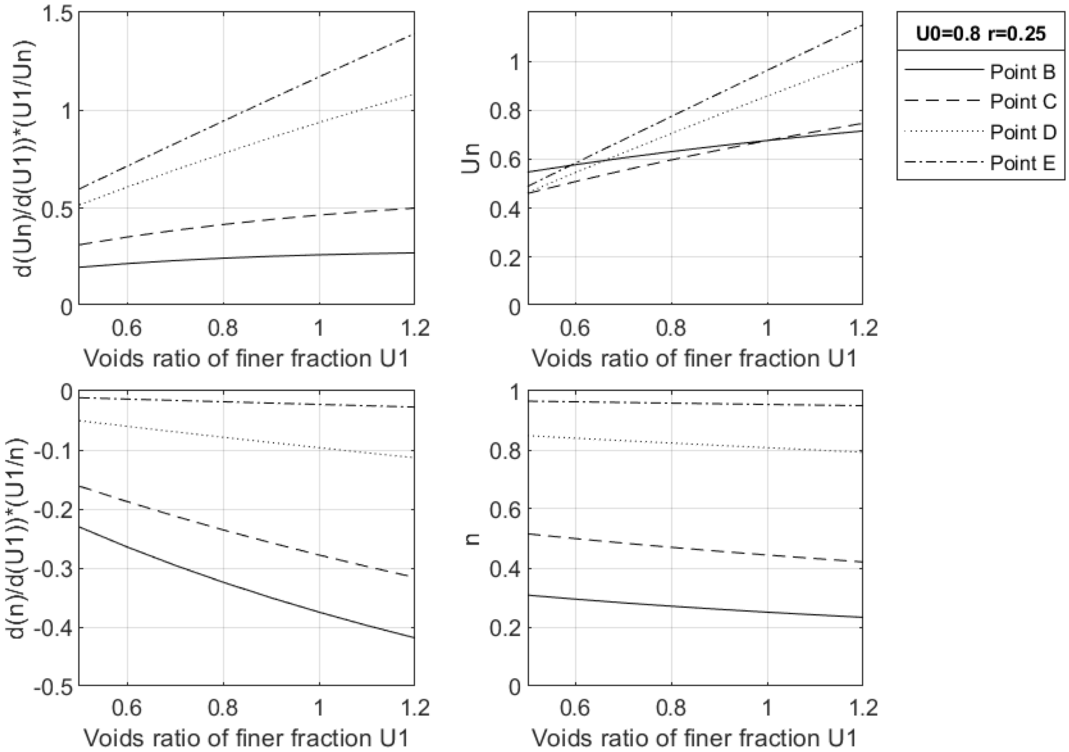

Figure 3: The sensitivity of U_n and n to the parameter U_1, at the changing points B, C, D and E, for the specific case with $U_0 = 0.8$, $r = 0.25$ and m, k_{int} and k_p according to the values mentioned in Table 1 for the changing points B, C, D and E

It can be seen from the Figure 3 that the sensitivity of the voids ratio of the mixtures (U_n) to the parameter U_1 increases when the voids ratio of fine fraction U_1 increases. Moreover, the sensitivity is higher for mixtures with a higher n value. A relative increase of U_1 leads to a relative increase of U_n. In contrast, a relative increase of U_1, leads to a relative decrease of n. The fine fraction value (n) of changing point D and E is not that sensitive to U_1 compared to that of the voids ratio U_n.

2.1.2 Voids Ratio of Coarse Fraction U_0

Due to complexity of the mathematical formulas, from now on, the results of the analysis are only presented in a graphical way. Figure 4 shows that the sensitivity of U_n and n to the parameter U_0, for U_0 values ranging from 0.5-1.2, keeps almost constant for each of the changing points (except B and C). Limited sensitivity can be noticed for point D and E from Figure 4. It could be speculated that when n is high, the U_n and n values of the changing points D and E will not deviate a lot, regardless of the voids ratio of coarse fractions (U_0). For changing point B and C, the voids ratio of mixes (U_n) and the fine fraction value (n) are easily to deviate for different voids ratio of the coarser fraction (U_0). Sensitivity coefficients of U_0 to both U_n and n are positive and a relative increase of U_0 leads thus to a relative increase of both U_n and n.

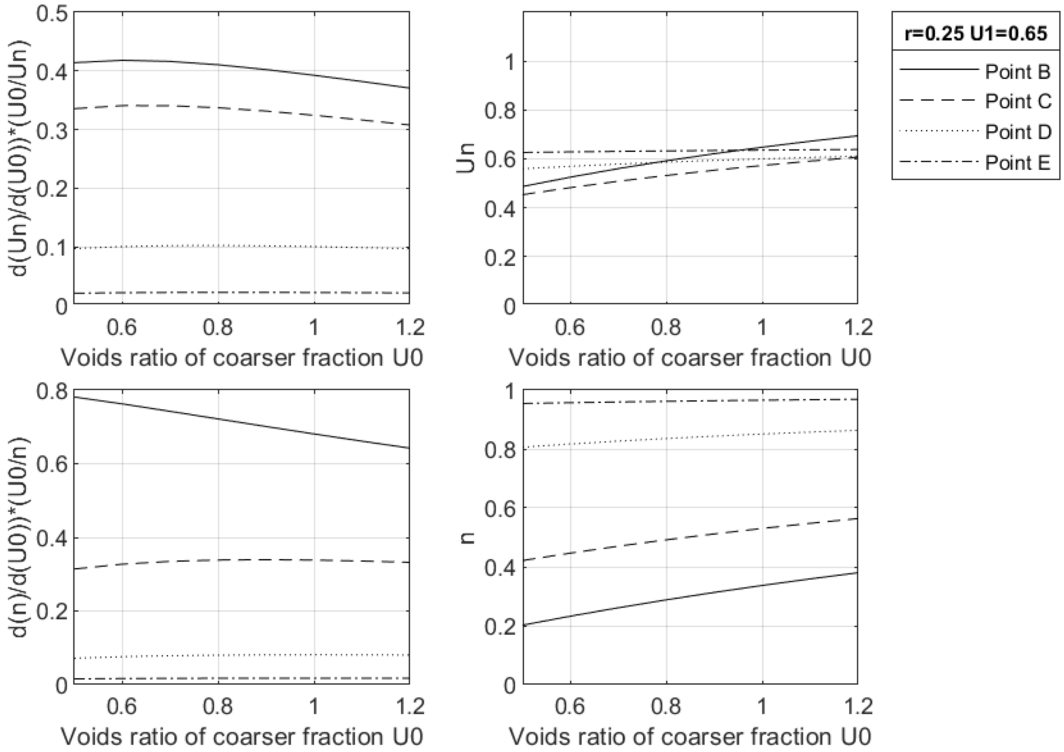

Figure 4: The sensitivity of U_n and n to the parameter U_0, at the changing points B, C, D and E for the specific case with $U_1 = 0.65$, r = 0.25 and m, k_{int} and k_p according to the values mentioned in Table 1 for the changing points B, C, D and E

2.1.3 Size Ratio r

Figure 5 illustrates the sensitivity in determining U_n and n to the size ratio (r) for each changing point. A relative increase of r leads to a relative increase of both U_n and n. Moreover, the changing points B and C have a high sensitivity in mid-range and high size ratio range while the changing point D and E are more sensitive in lower size ratio range, especially when r is smaller than roughly 0.5.

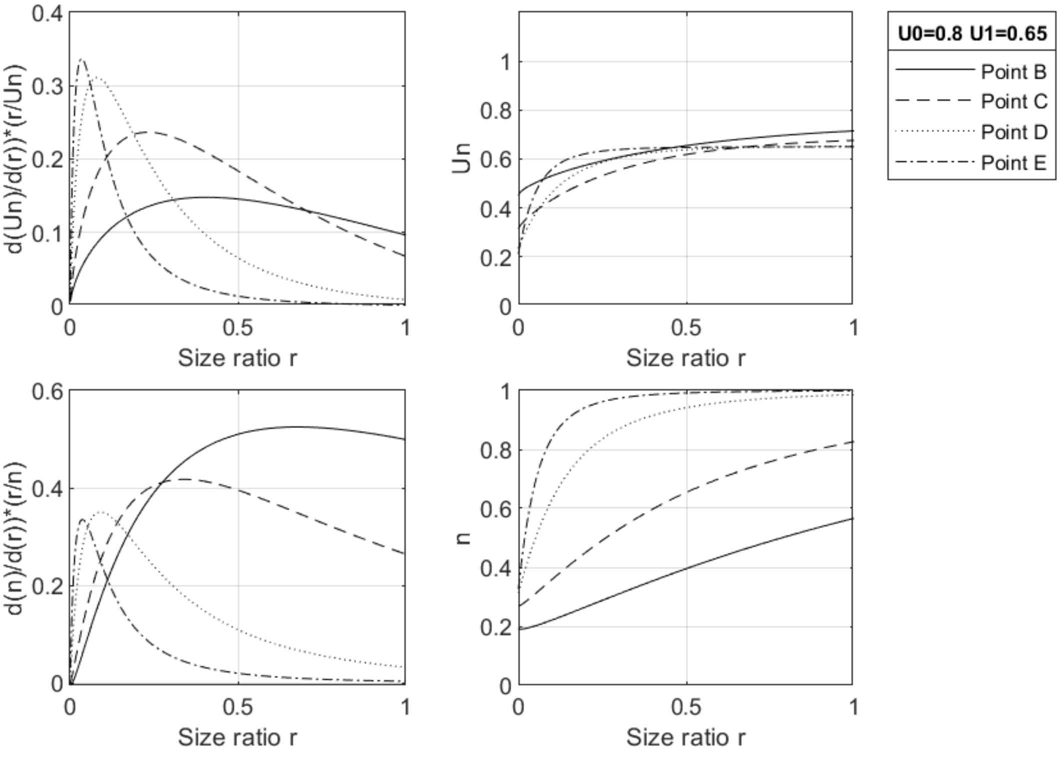

Figure 5: The sensitivity of U_n and n to the parameter r, at the changing points B, C, D and E for the specific case with U_0 = 0.8, U_1 = 0.65, r = 0.25 and m, k_{int} and k_p according to the values mentioned in Table 1 for the changing points B, C, D and E

2.1.4 Spacing Factor $\boldsymbol{m}$ and Empirical Factors $\boldsymbol{k_{int}}$ and $\boldsymbol{k_p}$

Figure 6, Figure 7 and Figure 8 give the sensitivity to parameters related with interaction factors in the determination of U_n and n at reasonable variations (taking into account the values defined by Dewar for each of the parameters at the changing points (Table 1)).

It can be seen from Figure 6 that the voids ratio of the mixes (U_n) and fine fraction values (n) of the changing points are quite sensitive to the interaction factor m. Compared with voids ratio of the mixes (U_n), the fine fraction value (n) of those points are more sensitive to m.

However, the empirical factors k_{int} and k_p, which are associated with the notional width factor Z, have almost no influence on determining the changing points (except changing point B in relation to k_{int}) for the voids ratio (U_n) and fine fraction value (n) of the mixes (Figure 7 and Figure 8). Figure 7 shows that the voids ratio of the mixes (U_n) and fine fraction value (n) in changing point B have high sensitivity to the empirical factor k_{int} and the sensitivity is higher for the voids ratio (U_n) rather than the fine fraction value (n).

The results obtained (Figure 6, Figure 7 and Figure 8) show that the m interaction factor has great influence on determining the changing points in the voids ratio diagram in the aspect of fine fraction value (n) rather than that of voids ratio (U_n), even when the n value is high (up to the fine fraction value at around 0.9). The wall effect is more explicit when the coarse fraction is intermediate present in the mix.

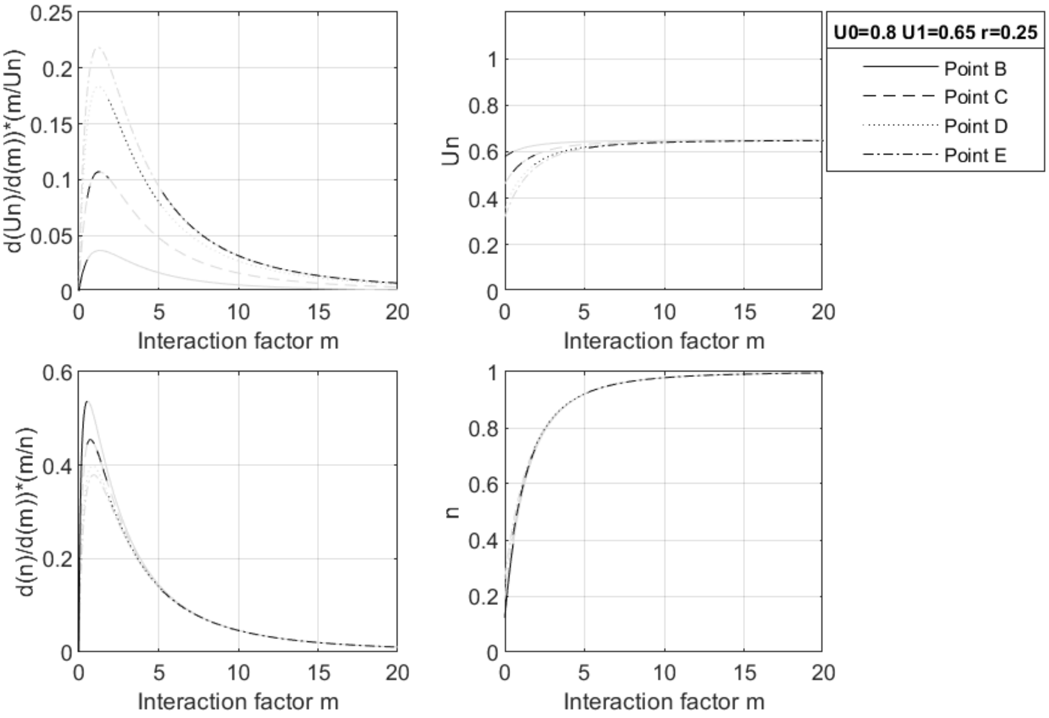

Figure 6: The sensitivity of U_n and n to the parameter m, at the changing points B, C, D and E for the specific case with U_0 = 0.8, U_1 = 0.65, r = 0.25 and k_{int}, k_p according to the values mentioned in Table 1 for the changing points B, C, D and E

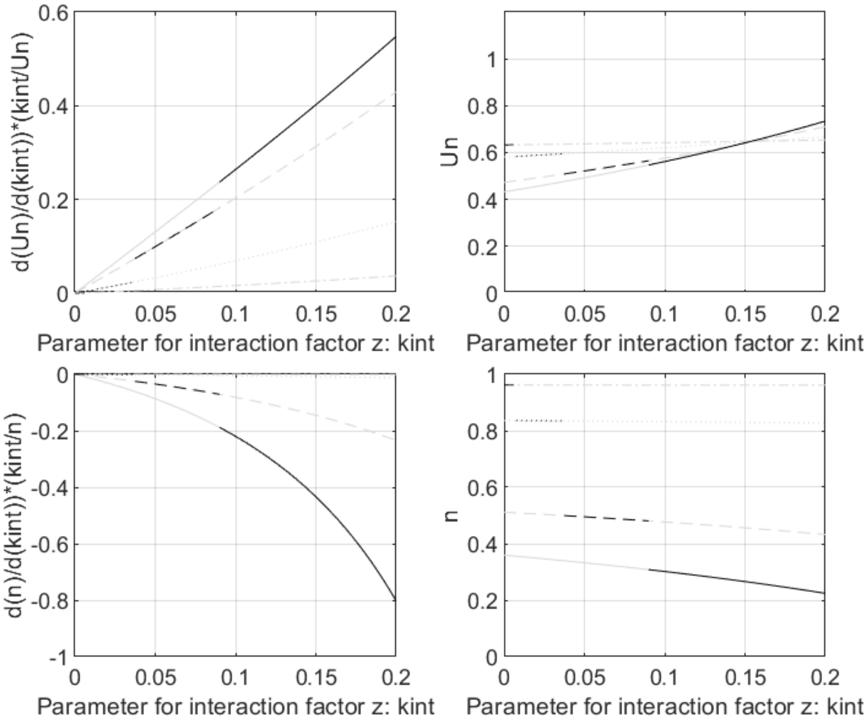

Figure 7: The sensitivity of U_n and n to the parameter k_{int}, at the changing points B, C, D and E, for the specific case with U_0 = 0.8, U_1 = 0.65, r=0.25 and m, k_p according to the values mentioned in Table 1 for the changing points B, C, D and E

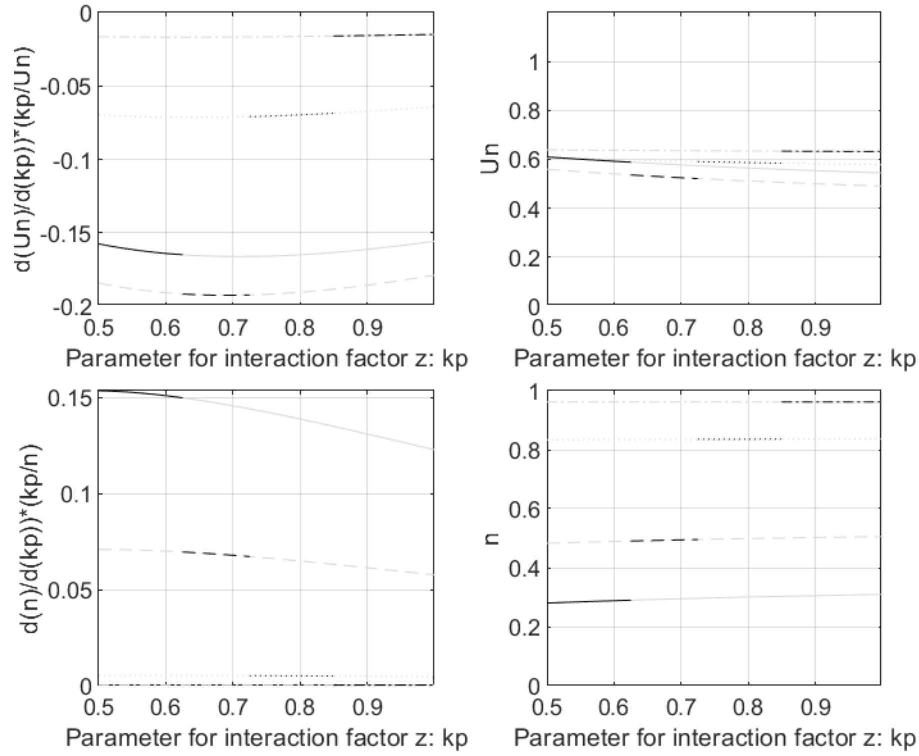

Figure 8: The sensitivity of U_n and n to the parameter k_p, at the changing points B, C, D and E, for the specific case with U_0 = 0.8, U_1 = 0.65, r=0.25 and m, k_{int} according to the values mentioned in Table 1 for the changing points B, C, D and E

3 CONCLUSIONS

This paper presents a sensitivity analysis towards parameters associated with the predicted voids ratio of two mixed fractions based on Dewar's packing theory. The sensitivity of U_n (voids ratio of the mix) and n (fine fraction ratio) towards one specific parameter has been graphically presented for a specific example in which the other parameters were kept constant and take the following values: U_1 = 0.65, U_0 = 0.8 , r = 0.25 and m, k_{int} and k_p according to the values mentioned in Table 1 for the changing points B, C, D and E. The results clearly show that:

- variation on U_0 has the highest impact on the U_n and n values of point B
- variation on U_1 has the highest impact on the U_n value of point E, while it has the highest impact on the n value of point B,
- for lower size ratios, U_n and n values of point D and E will mainly be affected upon a variation in r, while for mid-range and high r values U_n and n values of point B and C will mainly be affected.
- variation on m has a larger impact on the n value than on the U_n values of the changing points, even for high n values.
- the wall effect is more likely to appear for intermediate coarse fractions. Variations on k_{int} result in the variation of mixes' voids ratio (U_n) and fine fraction value (n), mainly at point B and C. Effect of variations of k_p on U_n and n is limited.

4 ACKNOWLEDGEMENTS

This work has been supported by a start-up funding (NO. STG/17/00290) from KU Leuven. This financial support is gratefully acknowledged.

5 REFERENCES

Dewar, J. D. (1999) ‘Computer Modelling of Concrete Mixtures’, in. E & FN Spon, London.