Steel and synthetic fibres for enhancing concrete toughness and shrinkage behaviour

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ABSTRACT: Fibre Reinforced Concrete (FRC) was a promising material a couple of decades ago but now is a reality. Fibres are now widely used for reinforcing industrial floors and for tunnel linings. In the beginning, steel fibres were used to enhance concrete toughness but, recently, different types of synthetic fibres were proposed in the market.

In the present paper, toughness of concrete reinforced with different types of steel and synthetic fibres was measured by means of bending tests on notched specimens. Furthermore, since fibres can be useful for limiting shrinkage cracking, early tests for comparing the efficiency of fibres in limiting the shrinkage cracks were performed. In addition, the benefits in terms of concrete toughness and shrinkage cracking of a combination of polypropylene and steel fibres are presented.

Finally, after determining the constitutive laws for cracked concrete, structural behaviour of slabs on grade and simply supported slabs was numerically simulated within the assumptions of Non Linear Fracture Mechanics.

1 INTRODUCTION

Although concrete technology allowed to increase its compressive strength, workability and durability moving towards High Performance Concrete (HPC), concrete tensile strength is still very low.

As a consequence, where tensile stresses are present, cracks form already under small loads and widen and propagate very quickly.

Moreover, cracks are present even before loading because of shrinkage phenomena. For this reasons, reinforcing bars are usually adopted to obtain reinforced concrete (r.c.) structures and represents a substantial part of the total costs because of the material costs, the labour costs (for placing) and the time needed for controlling the correct placement.

A valid alternative for the concrete reinforcement is represented by fibres. The basic idea from the application of discontinuous fibres in a concrete matrix is that they can bridge cracks, in a way somehow similar to conventional reinforcement.

Fibre Reinforced Concrete (FRC) was a promising material a couple of decades ago but now is a reality. Fibres are now widely used for reinforcing industrial floors and for tunnel linings.

In order to enhance concrete toughness (and ductility) for structural applications, high-modulus fibres can be used (Romualdi & Mandel 1964), were fibres aims to substitute, partially or totally, conventional reinforcement. Other types of fibres, having usually a low modulus and a small size (length of few millimetres and diameter of few microns) can be adopted to reduce shrinkage cracking and to enhance fire resistance (Balaguru & Shah 1992, di Prisco et al. 2004, Ahmad et al. 2004).

In the beginning, steel fibres were usually adopted to enhance concrete toughness but, recently, different types of synthetic macro-fibres (with a length of 20-50 mm) were proposed in the market.

Synthetic fibres have less problems for concrete pumpability and are not subjected to corrosion on the external surface that can provoke aesthetic problems.

A more recent development in fibre reinforced cement-based composites concerns the use of hybrid system of fibres (HyFRC) to contemporarily optimize several material performances (Banthia et al. 2000). In a HyFRC, two or more different types of fibres can be properly combined to produce a composite whose mechanical and physical performances take benefits from each of the individual fibres and from a possible synergistic response (Sorelli et al. 2004).

As an example, concrete toughness can be optimized by using fibres that will affect the cracking process during different stages of loading; this often involves fibres of varying sizes and materials (Sorelli et al. 2005, Markovic 2006).

Figure 1 shows the effects of micro- and macro-fibres on composite performance (Betterman et al. 1995); micro-fibres improve composite strength by bridging micro-cracks and, therefore, delaying the coalescence of these cracks into macro-cracks. On the other hand, macro-fibres are more effective in bridging larger crack openings (Banthia & Nandakumar 2003).

Therefore, HyFRC can be considered a multifunctional material that is able to achieve a set of desired performances by the use of different fibre types; as a second example, toughness and ductility can be provided by high-modulus fibres, shrinkage-cracking control by low-modulus fibres, conductivity by carbon fibres, etc. (Cominoli et al. 2005).

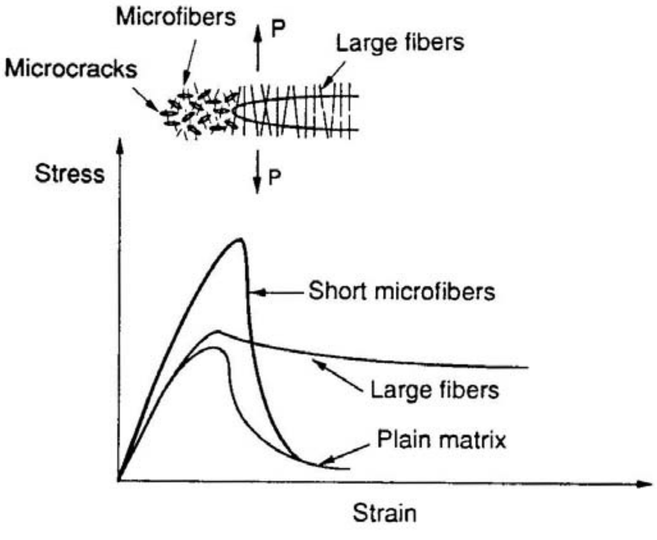


Figure 1. Effect of fibre size on crack bridging (Betterman et al. 1995).

In the present paper, an experimental program carried out to determine toughness and shrinkage behaviour of FRC with steel and synthetic fibres is presented. However, structural behaviour depends certainly on material properties but also on structural aspects and the degree of redundancy of the structure. The role played by the material and by the structural aspects was studied by performing FE analyses based on Non Linear Fracture Mechanics (Hillerborg et al. 1976) to better simulate the behaviour of FRC elements.

2 MATERIALS AND TEST SET-UP

Experiments were carried out on specimens made of a Normal Strength Concrete (C30/37) prepared with 340 kg/m^3 of cement (CEM II/A-LL 42.5 R according to UNI-EN 197), 190 kg/m^3 of water (water-cement ratio of 0.56), 6 l/m^3 of plasticizer and 1870 kg/m^3 of aggregates having a maximum size of 12 mm, as summarized in Table 1.

Two groups of fibres were considered in this research: the first concerns five different types of steel fibres while the second one regards four types of synthetic fibres; their geometrical and mechanical characteristics are reported in Tables 2 and 3.

Table 1. Concrete mixture proportions.

Portland Cement 42.5R	340	(kg/m^3)
Water	190	(l/m^3)
Acrylic plasticizer	6	(l/m^3)
Natural river aggregate	1450	(kg/m^3)
Crushed aggregate	420	(kg/m^3)
Filler calcareous	40	(kg/m^3)
Water-cement ratio	0.56	(-)

Table 2. Steel fibres geometry and characteristics.

Fibre	Steel-1	Steel-2	Steel-3	Steel-4	Steel-5
Geometry	Hooked	hooked	hooked	hooked	hooked
Length (mm)	50	40	40	37	40
Diameter (mm)	1.0	0.6	0.62	0.6	0.6
Aspect ratio (-)	50	67	65	62	67
Tensile strength (MPa)	1050	1300	1250	1250	1200

Fibres were added to the concrete matrix in different combinations; in particular, steel fibres were used with a dosage of 35 kg/m^3 (corresponding to a volume fraction, V_f, of 0.44%), while synthetic fibres were used with two dosages, equal to 4 kg/m^3 ($V_f \approx 0.44\%$) and 8 kg/m^3 ($V_f \approx 0.88\%$).

Table 3. Synthetic fibres characteristics.

Fibre	Poly-1	Poly-2	Poly-3	Poly-4
Length (mm)	40	35	20	54
Equivalent diameter (mm)	0.44	0.2÷0.25	0.18÷0.22	0.069
Aspect ratio (-)	90	150	100	782
Tensile strength (MPa)	620	600÷750	600÷750	600÷758
Density (kg/m^3)	920	910÷950	910÷950	910

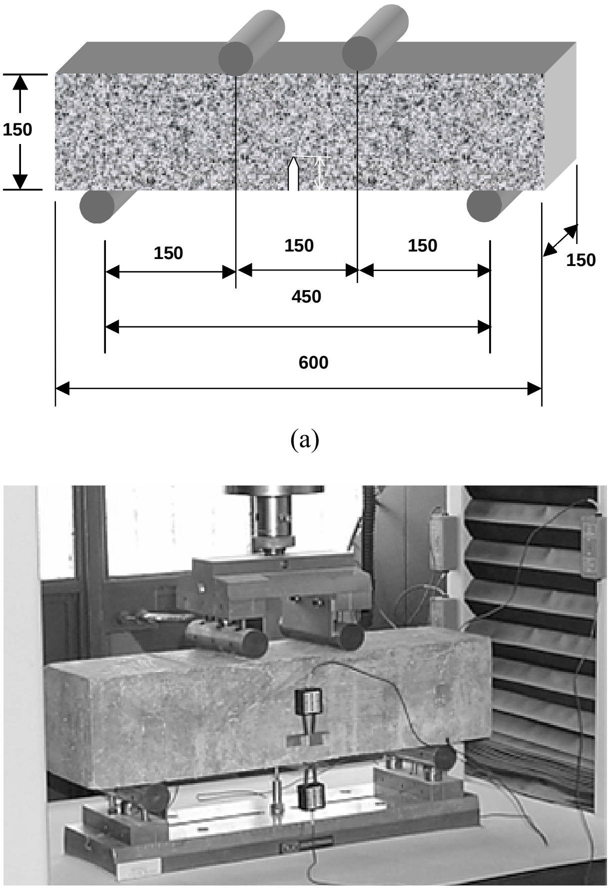
One HyFRC was also made by using steel fibres (0.38%) together with synthetic fibres (0.11%), as reported in Table 4 that also shows the compressive strength from cubes ($f_{c,cube}$) having a side of 100 mm, as determined after about 28 days of curing. The slump of the fresh concrete was always greater than 150 mm.

Eight beam specimens were cast for determining the fracture properties of each type of FRC. The latter were determined by using a 500 kN hydraulic testing machine (INSTRON-1274) with a closed loop control that permits to compensate the finite stiffness of the load system.

Four point bending tests were performed on notched beams having a size of 150x150x600 mm and a notch depth of 45 mm, according to the Italian Standard (UNI 11039; Fig. 2a). The Crack Mouth Opening Displacement (CMOD), measured by a Clip Gauge, was adopted as feedback signal. The Crack Tip Opening Displacement (CTOD) was also measured on both faces of the beam during the test (Fig. 2b).

Furthermore, early-age (< 24 h) deformation due to plastic shrinkage was measured. The specimens were thin plate (thickness equal to 20mm) with a section weakened by the presence of a hole and by wedges to better control the position of the shrinkage crack (Figs. 3 and 4).

The test set-up is rather simple and consists in measuring the cracks opening, by means of two mechanical gauges positioned across a weakened section of reduced thickness (20 mm; Fig. 3).



(a)

(b)

Figure 2. Geometry (a) and instrumentation (b) of the notched specimen for the bending tests, according to UNI 11039 (2003).

Table 4. Fibre combination, total volume fraction and compressive strength for the concrete.

Materials	Steel fibre (%)$_{vol.}$					Synthetic fibre (%)$_{vol.}$				Compression strength
	Steel-1	Steel-2	Steel-3	Steel-4	Steel-5	Poly-1	Poly-2	Poly-3	Poly-4	$f_{c,cube}$ (MPa)
SFRC	0.44	0.44	0.44	0.44	0.44	-	-	-	-	43.6
PFRC-A	-	-	-	-	-	0.44	0.44	0.44	0.44	40.9
PFRC-B	-	-	-	-	-	0.88	0.88	-	0.88	35.2
HyFRC	0.38	-	-	-	-	-	-	0.11	-	48.9

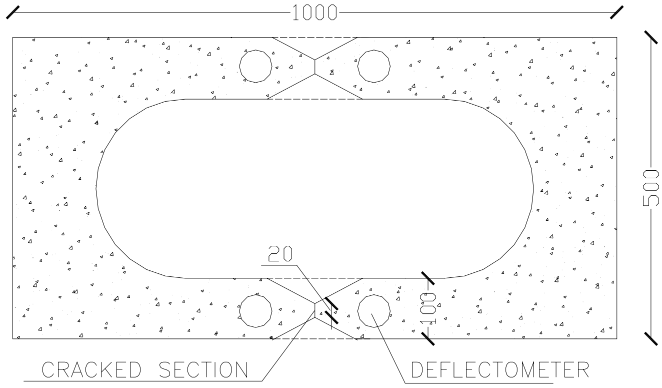


Figure 3. Geometry of a shrinkage specimen.

Figure 4. Particular of a shrinkage crack.

3 EXPERIMENTAL RESULTS

3.1 *Four point bending tests*

The experimental results obtained from tests on FRC specimens with different types of fibres, are summarized in Figures 5 and 6, in terms of load versus CTOD. It should be observed that all FRC specimens were characterized by the same volume fraction of fibres (0.44% for SFRC and PFRC-A in Fig. 5 and 0.88% for PFRC-B in Fig. 6); only HyFRC had a different volume fraction of fibres, equal to 0.49%.

For an easier comparison, the two figures show only one representative curve for each material, the one closest to the average curve. It can be observed that fibre geometry and material remarkably influence concrete toughness (evidenced by the post-peak stress) while the peak stress is not influenced by the presence of different fibres (it mainly depends on the tensile strength of the concrete matrix).

Synthetic micro-fibres slightly increase the post peak strength for small crack openings but this residual strength rapidly disappears since fibres are pulled out from the matrix. On the other hand, steel macro-fibres become efficient for larger crack openings. In particular, steel and hybrid fibres significantly improve the toughness of the material, while the polypropylene fibres (with the same V_f) have a lower effect (on concrete toughness).

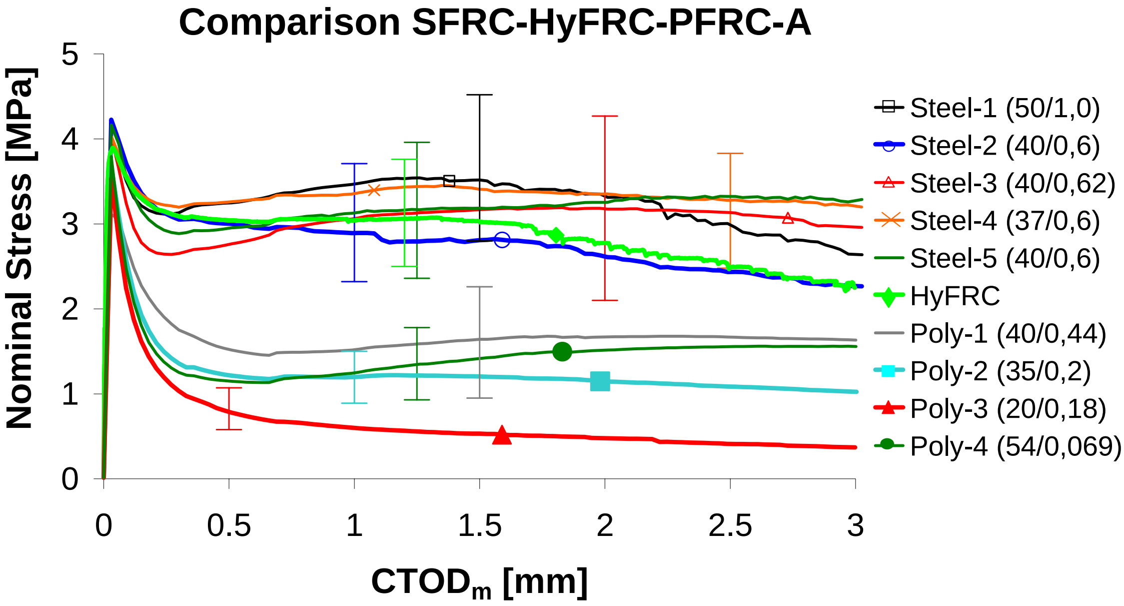


Figure 5. Nominal stress versus CTOD curves (average curves) obtained from bending tests on specimens reinforced with SFRC (V_f=0.44%), PFRC-A (V_f=0.44%) and HyFRC (V_f=0.49%).

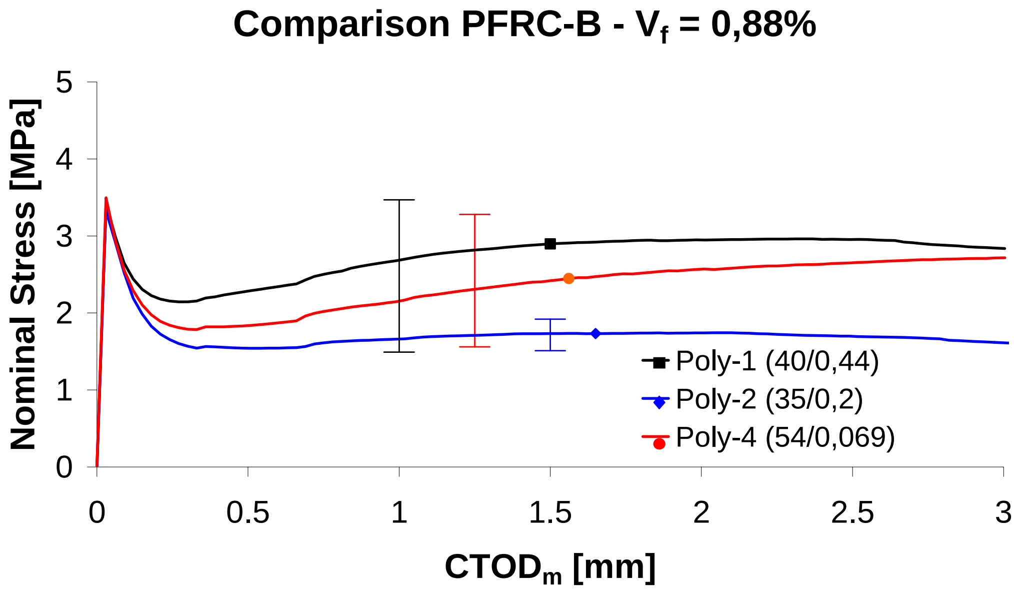


Figure 6. Nominal stress versus CTOD curves (average curves) obtained from bending tests on specimens reinforced with PFRC-B (V_f=0.88%).

Experimental results from FRC with low fibre contents are very sensitive to the number of fibres in the cracked sections which have a higher degree of variation in the smaller surface areas, especially when notched specimens are adopted (Sorelli et al. 2005).

Figure 7 reports the average value of the density of steel fibres in the cracked surface (number of fibres in the cracked surface per unit area) for each material, as well as the standard deviation (in brackets). Because of the large number, it was not possible to count the synthetic fibres in the cracked section. It can be noticed that the average density of steel fibres is around 0.6÷0.7 fibres/cm^2, and that the mean value of the scatter in the fibre density is 9.2%. The scatter of the experimental results markedly influences the characteristic and the design values of the material properties, much more than other structural materials (CNR 2006).

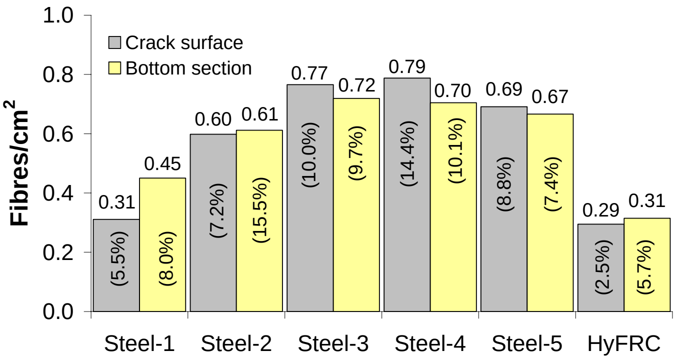


Figure 7. Density of steel fibres (average values) in the cracked section with the standard deviation (in brackets).

In order to better identify the post-cracking response of FRC, three independent parameters are proposed by the Italian Standard (UNI 11039, 2003), namely the first crack strength (f_{If}) and two equivalent flexural strengths ($f_{eq,(0-0.6)}$ and $f_{eq,(0.6-3)}$). The first flexural strength ($f_{eq,(0-0.6)}$) corresponds to a Crack Tip Opening Displacement (CTOD) range of 0-0.6 mm (significant for the Serviceability Limit State) while the second one corresponds to a CTOD range of 0.6-3 mm (significant for the Ultimate Limit State).

The equivalent flexural strengths obtained from bending tests on the FRC are given in Figure 8 (mean values) while the two ductility indexes D_0 (=$f_{eq,(0-0.6)}/f_{If}$) and D_1 (=$f_{eq,(0.6-3)}/f_{eq,(0-0.6)}$) are shown in Figure 9.

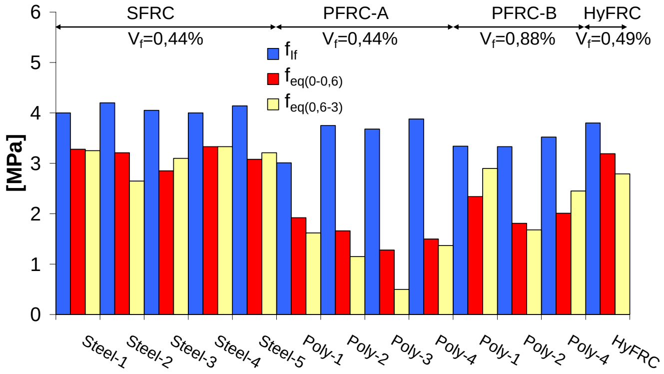


Figure 8. First-crack strength and post-cracking equivalent strengths according to UNI 11039 (2003).

However, the use of different fibres showed little or no improvement in first cracking point since the latter mainly depends on the concrete matrix (in fact, fibres start activating after cracking of the concrete matrix).

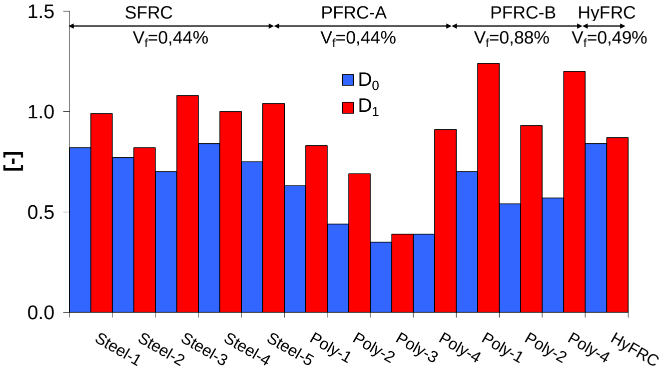


Figure 9. Ductility indexes according to UNI 11039 (2003).

One should observe that, when the same volume fraction of fibres was adopted, Steel Fibre Reinforced Concrete (SFRC) showed a higher toughness (i.e. higher post-cracking strength) as compared with Plastic Fibre Reinforced Concrete (PFRC). Toughness of PFRC was lower that that of SFRC even when doubling the volume fraction of synthetic fibres (Fig. 8). On the contrary, concrete with hybrid fibres showed a similar post-cracking strength of SFRC.

3.2 *Plastic shrinkage*

Shrinkage is the decrease with time of concrete volume. The decrease is due to changes in the moisture content of the concrete and physical-chemical changes, which occur without stress attributable to actions external to the concrete.

Early-age shrinkage due to evaporation (often referred to as plastic shrinkage) and the volume change related to hydration reactions have become reoccurring problem in concrete construction (Neville 2000).

If the shrinkage is non-uniform, or if there is restraint, tensile stresses develop which may result in cracking as the concrete has a low tensile strength and strain capacity at this stage. Moreover, it is not uncommon that these cracks run through the whole depth of the member (ACI 209 1992).

A further aim of the present research program concerned an early investigation on the influence of steel and synthetic fibres on the development of early-age cracks.

It is generally recognized the benefit of the synthetic micro-fibres to the reduction of the shrinkage cracks; what it is still unknown, is how quantifying this benefit in order to compare the performance of different fibres or the best volume fraction to adopt.

Since shrinkage cracking is a structural phenomenon, the geometry of specimens has a special influence. Due to the absence of a standardized test, the shrinkage test adopted in the present work aims to reproduce the severe condition of a thin slab with a large surface exposed to environmental conditions.

Two slabs for every material were tested and, in each slab, two different shrinkage cracks were observed. Tests mainly concerned PFRC slabs; however, one SFRC slab and one plain concrete slab were also tested to obtain reference values.

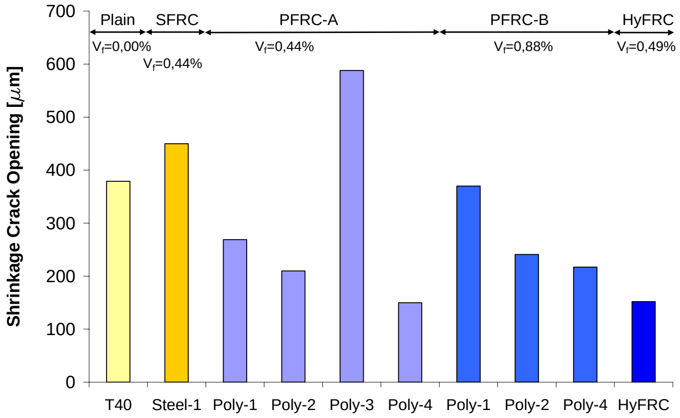


Figure 10. Experimentally shrinkage crack opening.

In order to reproduce more severe conditions, all the slabs have been submitted to an artificial ventilation in laboratory (by using a fan, with the same modalities for all the specimens), immediately after casting.

Figure 10 shows the average value of two different readings, after 20 hours of curing. It can be noticed that concretes reinforced with synthetic fibres (PFRC) showed lower cracks width than plain concrete and SFRC, with the only exception of FRC with 4 kg/m^3 of fibres Poly 3.

However, smaller cracks were also observed in the HyFRC slabs.

4 NUMERICAL SIMULATIONS OF 4PBT

Inverse analyses of the bending tests allowed to determine the best-fitting softening law for the FRCs adopted in the present research (Roelfstra & Wittmann 1986).

The experimental bending tests were simulated by adopting Non Linear Fracture Mechanics (NLFM, Hillerborg et al. 1976) with MERLIN (Reich et al. 1994) which is based on a discrete crack approach.

The aim of the numerical simulations was to define a stress-crack opening law (σ-w) for the post cracking behaviour of each material.

Fracture of plain concrete or concrete reinforced with a single type of fibre can be approximated with a bilinear law where the first steeper branch simulates the bridging of the early micro-cracks while the second branch simulates the aggregate interlocking in plain concrete or the fibre links in FRC (Wittmann et al. 1988).

When two types of fibres are adopted, a trilinear law may better represent the post-cracking behaviour since different fibres may activate at different crack openings. In the present work, a trilinear law was used for HyFRC where synthetic fibres were added to steel fibres.

The Finite Element mesh for the beam tests was based on 2348 elastic triangular elements (plane stress) and 34 interface elements (with zero thickness) in the fracture sections (whose position in known because of the notch) to simulate a fictitious crack (Fig. 11).

In the numerical analyses, the modulus of elasticity (33350 MPa) and the tensile strength (3.09 MPa) were determined from the compressive strength, in accordance with Eurocode 2 (2003).

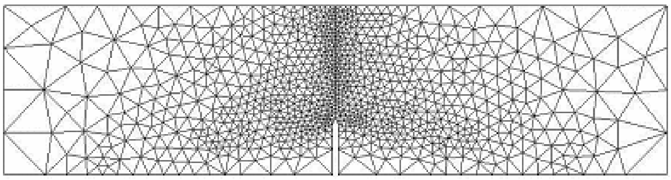
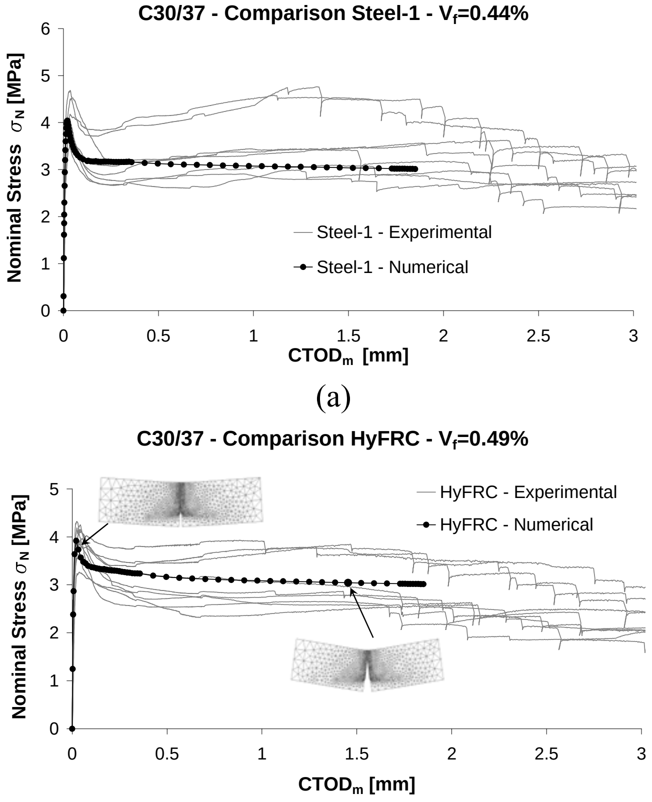


Figure 11. Mesh adopted for the numerical simulations of the beam tests based on a discrete crack approach.

Table 5. Constitutive parameters for the σ-w law.

Mat.	f_{ct} (MPa)	σ_1 (MPa)	w_1 (mm)	σ_2 (MPa)	w_2 (mm)	w_{cr} (mm)
Steel-1	3.09	1.100	0.022	-	-	7.20
Steel-2	3.09	1.186	0.021	-	-	4.16
Steel-3	3.09	0.927	0.024	-	-	4.56
Steel-4	3.09	1.100	0.022	-	-	6.30
Steel-5	3.09	1.100	0.022	-	-	5.39
Poly-1	3.09	1.186	0.021	-	-	0.28
Poly-2	3.09	0.668	0.027	-	-	1.08
Poly-3	3.09	1.186	0.021	-	-	0.20
Poly-4	3.09	0.841	0.025	-	-	0.74
HyFRC	3.09	1.186	0.021	1.057	0.30	7.20



(a)

(b)

Figure 12 – Experimental and numerical curves of the nominal stress (σ_N) versus crack tip opening displacement (CTOD$_m$).

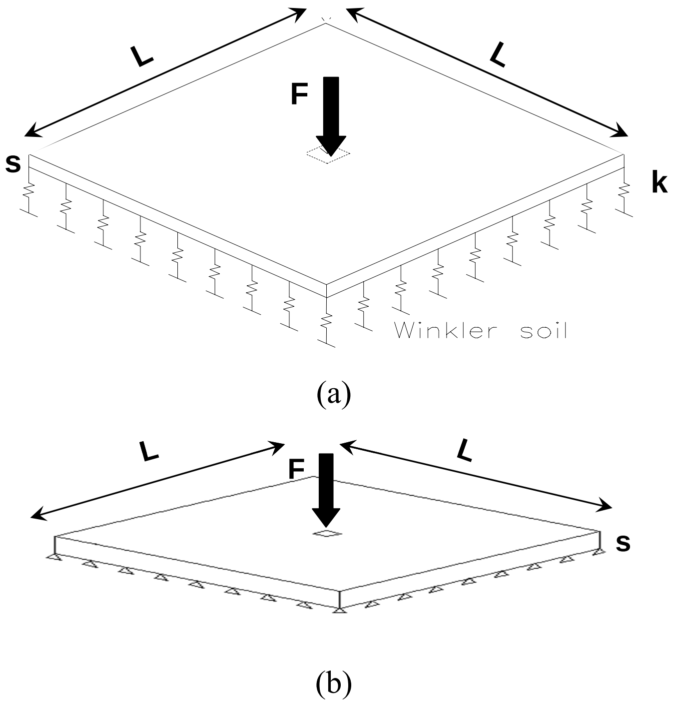
The parameters of the poly-linear laws that provided the best fit for the experimental curves are summarized in Table 5.

Typical numerical curves of the nominal stress (σ_N) versus CTOD$_m$ from bending tests are plotted with the experimental curves in Figure 11. It can be noticed that a bilinear softening law provides a good approximation of the experimental results from a FRC with a single fibre (Fig. 12a). As expected, concrete reinforced with hybrid-fibres is better approximated by a tri-linear curve since synthetic fibres start activating before the steel ones (Fig. 12b).

5 FROM MATERIAL TO STRUCTURE

The behaviour of a FRC structure is markedly influenced by the characteristics of the material but also by structural geometry and its degree o redundancy. Due to the high toughness of FRC and the stress redistribution in statically undetermined structures (as slabs on ground), the ultimate load is remarkably higher than the first-crack load (Falkner et al. 1995).

The material properties of the different FRC$_s$ (Tab. 5) were used to predict the mechanical behaviour of slabs on grade (Fig. 13a) and of slabs simply supported along the edges (Fig. 13b). The subgrade was approximated as a Winkler soil.



(a)

(b)

Figure 13. Slab on a Winkler soil (a) and supported on the edges (b).

The analyses were performed under the assumptions of Non Linear Fracture Mechanics (Hillerborg et al. 1976) to better predict the crack development until a collapse mechanism occurs (Meda & Plizzari 2004). The geometrical properties and the Winkler constant (*k*) are shown in Table 6.

Table 6. Geometrical parameters of the slabs.

L	S	k	A
(mm)	(mm)	(N/mm^3)	(mm^2)
4000	150	0.12	380 x 380

Slab behaviour was numerically simulated by means of MERLIN (Reich et al. 1994); Figure 14 evidences the adopted FE mesh.

Figure 15 shows the numerical curves of the load versus the displacement (slab central point), obtained from the slabs on grade made with the tested FRCs. The numerical curves show the significant load increase after the first crack appears in the slab (Falkner et al. 1995).

The response of slabs reinforced with synthetic fibres show a slightly smaller ultimate load than SFRC slabs because of the high degree of redundancy of the slab on grade; in addition, these fibres can be useful for better control shrinkage cracking. The slab with hybrid fibres behaves very similarly to SFRC slabs.

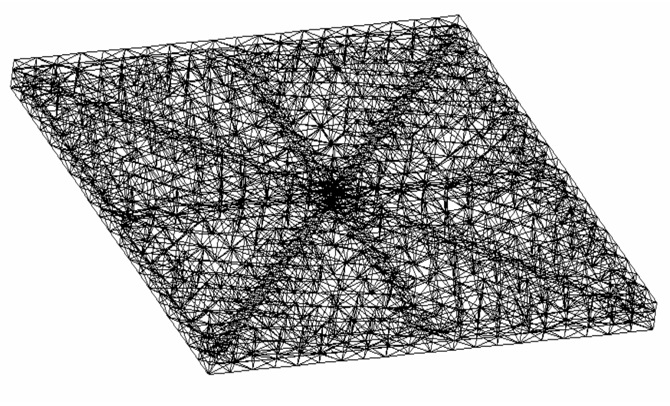


Figure 14. F.E. Mesh of the slab.

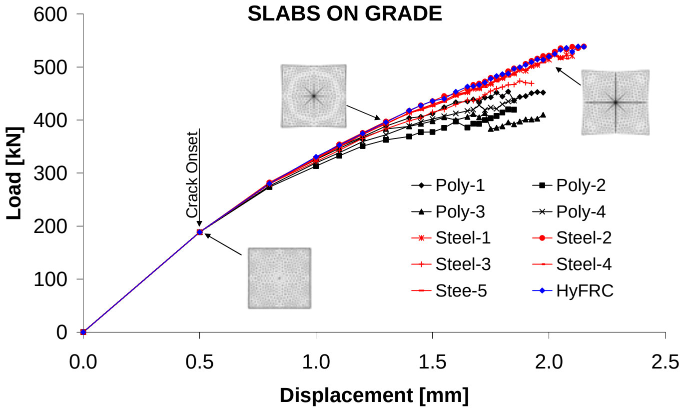


Figure 15. Numerical load versus vertical displacement measured at the centre of the slab on grade.

The numerical curves of the slabs supported along the edges are shown in Figure 16 in terms of load versus vertical displacement of the central point of the slab. For these boundary conditions, the FEA results show a greater difference between synthetic, steel and hybrid elements with respect to slabs on grade; this is due to the lower degree of redundancy of these structures with respect to slabs on elastic ground.

It can be also observed that the load is reduced after the peak and that the ultimate load (post-peak) of the slabs reinforced with synthetic fibre is considerably lower than that of the slabs reinforced only with steel fibre. Once again, the slab with hybrid fibres exhibits a behaviour similar to SFRC slabs, but with the advantage of a better control of the shrinkage cracks (because of the presence of polypropylene micro-fibres). These numerical results underline the need of adding conventional reinforcement in slabs made of FRC with a softening behaviour that generally occurs with volume fraction of fibres lower than 2%.

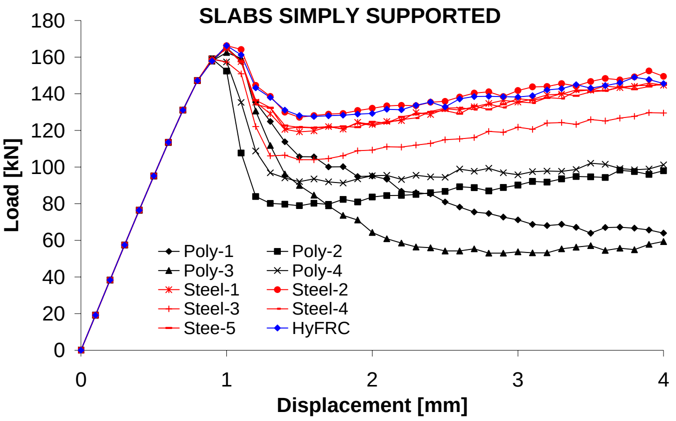


Figure 16. Numerical load versus vertical displacement measured at the centre of the slab supported along the edges.

6 CONCLUDING REMARKS

Experimental results from fracture and shrinkage tests carried out on a normal strength concrete reinforced with steel, synthetic and hybrid fibres are presented herein. The fibres have different geometries and are also combined to obtain hybrid composites.

Bending tests were performed on notched specimens to determine the material properties, while early shrinkage tests aimed to reproduce the environmental conditions of a thin slab (thickness of 20 mm) with a surface exposed to the air (i.e. the external slab of a precast panel).

The main experimental results showed the better behaviour of concrete reinforced with steel fibres in terms of toughness, while FRC with synthetic fibre showed a good behaviour in terms of reduction of shrinkage cracks.

A combination of steel and synthetic fibres can improve concrete toughness for both small and large crack opening displacements and reduce the width of shrinkage cracks.

A numerical simulation based on Non Linear Fracture Mechanics of the bending tests was carried out in order to gain a better understanding of the material properties in terms of post-cracking response (σ-w).

The numerical results show that the post-cracking behaviour can be approximated by means of a bilinear law when a single type of fibre is adopted; whereas FRC with hybrid fibres is better approximated by a poly-linear law that takes into account the different behaviour of fibres.

Structural behaviour of FRC elements depends on FRC properties but also on the structural degree of redundancy. When this degree is high (that is the case of slabs on grade) the large differences in material behaviour are not present in the structure; different is the case of structures with a lower degree of redundancy where FRCs with low fibre contents may need the addition of conventional reinforcement.

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